



The effect of the back button in a random walk: application for pagerank

Fabien Mathieu, Mohamed Bouklit

► To cite this version:

Fabien Mathieu, Mohamed Bouklit. The effect of the back button in a random walk: application for pagerank. WWW '04 - Special interest tracks and posters of the 13th international conference on World Wide Web, May 2004, New York, United States. pp.370-371, 10.1145/1013367.1013480 . hal-00668339

HAL Id: hal-00668339

<https://inria.hal.science/hal-00668339>

Submitted on 9 Feb 2012

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

The Effect of the *Back* Button in a Random Walk: Application for PageRank

Fabien Mathieu
Gyroweb – INRIA, LIRMM
34392 Montpellier Cedex 5 France
fmathieu@clipper.ens.fr

Mohamed Bouklit
LIRMM
34392 Montpellier Cedex 5 France
bouklit@lirmm.fr

ABSTRACT

Theoretical analysis of the Web graph is often used to improve the efficiency of search engines. The PageRank algorithm, proposed by [5], is used by the Google search engine [4] to improve the results of the queries.

The purpose of this article is to describe an enhanced version of the algorithm using a realistic model for the *back* button. We introduce a limited history stack model (you cannot click more than m times in a row), and show that when $m = 1$, the computation of this *Back* PageRank can be as fast as that of a standard PageRank.

Categories and Subject Descriptors

F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems—Computations on matrices

General Terms

Algorithms, Measurement

Keywords

Web analysis, PageRank, Random walk, flow, back button

1. INTRODUCTION

Since the introduction of the *PageRank* algorithm in 1998, numerous enhancement were made in both implementation and theoretical efficiency. Using the stochastic aspect of the PageRank algorithm, the concept of *backoff process* was introduced by Fagin *et al.* [3] as an idealized model of browsing the web using both hyperlinks and the *back* button. This model allow the history stack to grow unboundedly. We introduce a bounded history stack, and show that in the special case of a one page history, there is an explicit and fast algorithm for computing the PageRank.

2. NOTATIONS

Let $G = (V, E)$ be a web graph, that is a set V of web pages linked to each other by a set E of edges.

If G is aperiodic and strongly connected, it is well known [6] that the iterative process

$$\forall v \in V, n \in \mathbb{N}, P_{n+1}(v) = \sum_{w \rightarrow v} \frac{P_n(w)}{d(w)}, \quad (1)$$

where $d(v)$ is the out-degree of $v \in V$, converges towards an unique probability P for any given probability P_0 .

However the web graph is far from being strongly connected [2]. One solution is to introduce a dumping factor d . The principle of the dumping factor is to “dump” the iterative process:

$$\forall v \in V, n \in \mathbb{N}, P_{n+1}(v) = d \sum_{w \rightarrow v} \frac{P_n(w)}{d(w)} + (1-d)S(v), \quad (2)$$

where S is a given probability on V^1 .

A dumping factor is equivalent to working on a weighted strongly connected graph. If G is leafless, the limit P of (2) exists. Otherwise, normalization is needed.

3. BACK BUTTON MODEL

We suggest to refine the PageRank model by inserting the possibility to *return*. We choose a bounded history stack, so the PageRank algorithm is equivalent to a Markov chain with finite memory m . Potentially, this leads to consider all the possible paths in G of length m . For $m = 1$, this corresponds to the set E of the hyperlinks. We introduce two intuitive models for $m = 1$, one of them collapsing the working space from E to V . To begin with and for simplicity, we examine our *Back* button process without dumping.

3.1 Reversible *back*

In this model, we suppose that the web user can click at each state either on the links or on the *Back* button (the *Back* button is then considered as an outgoing link like the others). The probability of using the *Back* button is the same as that of using a given link. using *Back* button brings the user back to the previous state².

Let $P_n^{rb}(w, v)$ be the probability of being in v in the instant n coming from w in the instant $n - 1$. $P_n^{rb}(w, v)$ is defined if $(w, v) \in E$ or $(v, w) \in E$. We can express the probability $P_n(v)$ of being in v at the instant n as follows:

$$P_n(v) = \sum_{w \leftrightarrow v} P_n^{rb}(w, v) \quad (3)$$

Note that because of the *Back* process, we work on the non-directed graph induced by G .

Working on the same principle, we deduce an equation expressing $P_n^{rb}(w, v)$: if $(w, v) \notin E$ (but (v, w) is), going from w to v implies using the *Back* button; then we were previously in w coming from v . Otherwise, either the *Back* button or the regular link can be used. Thus we have:

¹Most of the time, $S \equiv \frac{1}{|V|}$, but some have suggested that it would be better to “personalize” it [1].

²Thus two consecutive uses of the *Back* button cancel each other.

$$P_{n+1}^{rb}(w, v) = \begin{cases} \frac{1}{d(w)+1} (P_n(w) + P_n^{rb}(v, w)) & \text{if } (w, v) \in E, \\ \frac{P_n^{rb}(v, w)}{d(w)+1} & \text{otherwise.} \end{cases} \quad (4)$$

Using (3) and (4) gives an iterative process for computing the new PageRank, but if $G' = (V, E')$ is the non-oriented graph induced by G , we have to use $|V| + |E'|$ variables instead of $|V|$ for the standard PageRank.

3.2 Irreversible Back

We now consider that the *Back* button cannot be used twice consecutively. This model, which seems more complex, as however three important advantages. First, it significantly reduces the stored PageRank by “greenhouse effect” in the end-nodes. Second, it is more appropriate to the insertion of a dumping factor (see 3.3). Finally it is less heavy on resource.

For $(w, v) \in E$, let $P_n^{ib}(w, v)$ be the probability to arrive at v using an hyperlink in w , and $\bar{P}_n^{ib}(v)$ the probability to arrive at v using the *Back* button. \bar{P}_{n+1}^{ib} can be deduced from P_n^{ib} :

$$\bar{P}_{n+1}^{ib}(v) = \sum_{w \leftarrow v} \frac{P_n^{ib}(v, w)}{d(w)+1} \quad (5)$$

Then we can tell P_{n+1}^{ib} from P_n^{ib} and \bar{P}_n^{ib} :

$$P_{n+1}^{ib}(w, v) = \frac{1}{d(w)+1} \sum_{u \rightarrow w} P_n^{ib}(u, w) + \frac{\bar{P}_n^{ib}(w)}{d(w)} \quad (6)$$

We can note that $P_{n+1}^{ib}(w, v)$ does not depend on the arrival node v . We can then use P_n^{ib} on V instead of E , specifying only the departure node.

Equations (5) and (6) can now be written:

$$\bar{P}_{n+1}^{ib}(v) = P_n^{ib}(v) \sum_{w \leftarrow v} \frac{1}{d(w)+1} \quad (7)$$

$$P_{n+1}^{ib}(v) = \frac{1}{d(v)+1} \sum_{w \rightarrow v} P_n^{ib}(w) + \frac{\bar{P}_n^{ib}(v)}{d(v)} \quad (8)$$

3.3 Back button and dumping

For a real graph, insertion of the *Back* button ensures there is virtually no leaf, but the process may still not be irreducible, so we need to introduce a dumping factor. We made the choice to deactivate the *back* button after a crossing³. We can then merge (2), (7) and (8) to obtain:

$$\bar{P}_{n+1}^{ib}(v) = dP_n^{ib}(v) \left(\sum_{w \leftarrow v} \frac{1}{d(w)+1} \right) + (1-d)S(v) \quad (9)$$

$$P_{n+1}^{ib}(v) = d \left(\frac{1}{d(v)+1} \sum_{w \rightarrow v} P_n^{ib}(w) + \frac{\bar{P}_n^{ib}(v)}{d(v)} \right) \quad (10)$$

³Deactivating the *back* button after a crossing avoids to consider the $V \times V$ crossing transitions in the *Back* process.

4. EFFECTIVE COMPUTATION

4.1 Convergence

The process we made is stochastic (there is no blind way), aperiodic and irreducible (because of the dumping factor). The Perron-Frobenius theorem applies and ensures that the iterative process converges towards a unique fixed point.

4.2 Optimization

Using (9) and (10), we get an iterative way of calculating P_n^{ib} , and $P_{n+1}^{ib}(v)$ is equal to:

$$\sum_{w \rightarrow v} \frac{dP_n^{ib}(w)}{d(v)+1} + \sum_{w \leftarrow v} \frac{d^2 P_{n-1}^{ib}(v)}{d(v)(d(w)+1)} + \frac{d(1-d)S(v)}{d(v)} \quad (11)$$

Equation (11) is a two terms recurrence, but as we want to compute a fix point, the Gauss-Seidel method allows to use P_n^{ib} instead of P_{n-1}^{ib} ; indeed one can approximate $P_{n+1}^{ib}(v)$ by:

$$\sum_{w \rightarrow v} \frac{dP_n^{ib}(w)}{d(v)+1} + \sum_{w \leftarrow v} \frac{d^2 P_n^{ib}(v)}{d(v)(d(w)+1)} + \frac{d(1-d)S(v)}{d(v)} \quad (12)$$

We remark that this iterative process has the same complexity that the standard PageRank computation.

Once P_n^{ib} has converged toward a vector P^{ib} , we obtain easily the asymptotic probability of presence P as follows:

$$P(v) = \sum_{w \rightarrow v} P^{ib}(w) + \bar{P}^{ib}(v) \quad (13)$$

5. CONCLUSION

We have proposed an alternative PageRank that can be obtained as easily that the standard PageRank and that should offer a better modelization of the web users. Computations made on a 8 millions pages graph showed that the top ranked pages differ from one model to another, yet both seemed interesting. We still have to merge this algorithm with a semantic pertinence-sort to be able to test this new model in the “real life”.

6. REFERENCES

- [1] S. Brin, R. Motwani, L. Page, and T. Winograd. What can you do with a Web in your Pocket? *Data Engineering Bulletin*, 21(2):37–47, 1998.
- [2] A. Broder, R. Kumar, F. Maghoul, P. Raghavan, S. Rajagopalan, R. Stata, A. Tomkins, and J. Wiener. Graph structure in the web. *Computer Networks: The International Journal of Computer and Telecommunications Networking*, 33(1-6):309–320, 2000.
- [3] R. Fagin, A. R. Karlin, J. Kleinberg, P. Raghavan, S. Rajagopalan, R. Rubinfeld, M. Sudan, and A. Tomkins. Random walks with “back buttons” (extended abstract). pages 484–493, 2000.
- [4] Google. <http://www.google.com/>, 1998.
- [5] L. Page, S. Brin, R. Motwani, and T. Winograd. The PageRank Citation Ranking: Bringing Order to the Web. Technical report, Computer Science Department, Stanford University, 1998.
- [6] L. Saloff-Coste. Lectures on finite Markov chains. In G. G. E. Giné and L. Saloff-Coste, editors, *Lecture Notes on Probability Theory and Statistics*, number 1665 in LNM, pages 301–413. Springer Verlag, 1996.